

Dirac-Neutrino Magnetic Moment and the Dynamics of a Supernova Explosion

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Abstract

The double conversion of the neutrino helicity $\nu_L \rightarrow \nu_R \rightarrow \nu_L$ has been analyzed for supernova conditions, where the first stage is due to the interaction of the neutrino magnetic moment with plasma electrons and protons in the supernova core, and the second stage, due to the resonance spin flip of the neutrino in the magnetic field of the supernova envelope. It is shown that, in the presence of the neutrino magnetic moment in the range $10^{-13} \mu_B < \mu_\nu < 10^{-12} \mu_B$ and a magnetic field of $\sim 10^{13}$ G between the neutrinosphere and the shock-stagnation region, an additional energy of about 10^{51} erg, which is sufficient for a supernova explosion, can be injected into this region during a typical shock-stagnation time.

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Numerical simulations of a supernova explosion encounter two main obstacles [1–5]. First, a mechanism stimulating the damping shock, which is likely necessary for the explosion, has not yet been well developed. Recall that shock damping is mainly due to energy losses on the dissociation of nuclei. Second, the energy release of the “theoretical” supernova explosion is much lower than the observed kinetic energy $\sim 10^{51}$ erg of an envelope. This is called the FOE (ten to the Fifty One Ergs) problem. It is believed that a self-consistent description of the explosion dynamics requires an energy of $\sim 10^{51}$ erg to be transferred via some mechanism from the neutrino flux emitted from the supernova central region to the envelope.

Dar [6] proposed a possible way for solving the above-mentioned problems. His mechanism is based on the assumption that the neutrino has a magnetic moment that is not very small. Left-handed electron neutrinos ν_e intensely generated in the collapsing supernova core are partially converted into right-handed neutrinos due to the interaction of the neutrino magnetic moment with plasma electrons and protons. In turn, the right-handed neutrinos sterile with respect to weak interactions freely leave the central part of the supernova if

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the neutrino magnetic moment is not too large, $\mu_\nu < 10^{-11} \mu_B$, where μ_B is the Bohr magneton. Some of these neutrinos can be inversely converted to left-handed neutrinos due to the interaction of the neutrino magnetic moment with the magnetic field in the supernova envelope. According to contemporary views, the magnetic field in this region can be up to about the critical magnetic field $B_e = m_e^2/e \simeq 4.41 \times 10^{13} \text{ G}$ ¹ or even higher [7–9]. The born again left-handed neutrinos can transfer additional energy to the envelope by virtue of the beta-type absorption $\nu_e n \rightarrow e^- p$.

In our opinion, the mechanism of the double conversion of neutrino helicity should be analyzed more carefully. It was shown in our recent work [10] that the flux and luminosity of right-handed neutrinos from the central region of the supernova were significantly underestimated in previous works. Here, we reconsider the process $\nu_L \rightarrow \nu_R \rightarrow \nu_L$ under supernova conditions and analyze the possibilities for stimulating the damping shock.

The neutrino spin flip $\nu_L \rightarrow \nu_R$ under physical conditions corresponding to the central region of the supernova has been studied in a number of works (see, e.g., [11–13]; a more extended reference list is given in [10]). The process is possible due to the interaction of the Dirac-neutrino magnetic moment with a virtual plasmon, which can be both generated and absorbed:

$$\nu_L \rightarrow \nu_R + \gamma^*, \quad \nu_L + \gamma^* \rightarrow \nu_R. \quad (1)$$

In [11], the neutrino spin flip was described in terms of scattering by plasma electrons and protons ($\nu_L e^- \rightarrow \nu_R e^-$ and $\nu_L p \rightarrow \nu_R p$, respectively) in a supernova core immediately after the collapse. However, the important polarization effects of the plasma on the photon propagator were not considered in that work. Instead, the photon dispersion was taken into account phenomenologically by introducing the so-called thermal mass of a photon into the propagator. The above-mentioned effects were analyzed more consistently in [12, 13], where the effect of a high-density astrophysical plasma on the photon propagator was taken into account using the thermal field-theory formalism. However, an analysis of works [12, 13] showed that they concerned only the electron component of the plasma, namely, only the channel $\nu_L e^- \rightarrow \nu_R e^-$, and only the electron contribution to the photon propagator, whereas the proton component of the plasma was not analyzed at all. This seems to be even stranger because the plasma-electron and proton contributions to the neutrino spin flip are of the same order according to [11].

A consistent analysis of processes (1), with neutrino-helicity conversion due to the interaction with both plasma electrons and protons via a virtual plasmon and with the inclusion of polarization effects of the plasma on the photon propagator was given in [10]. In particular, according to the numerical analysis, the contribution of the proton component of the plasma is not merely significant, but even dominant. As a result, using the data on supernova *SN1987A*, a new astrophysical limit was imposed on the electron-neutrino magnetic moment:

¹Hereafter, we use the natural system of units in which $c = \hbar = 1$, and $e > 0$ is the elementary charge.

$$\mu_\nu < (0.7 - 1.5) \times 10^{-12} \mu_B, \quad (2)$$

This is a factor of two better than previous constraints.

In particular, the function $\Gamma_{\nu_R}(E)$ determining the energy spectrum of right-handed neutrinos was calculated in [10]. In other words, this function specifies the number of right-handed neutrinos emitted per 1 MeV of the neutrino energy spectrum per unit time from unit volume of the central region of a supernova:

$$\frac{dn_{\nu_R}}{dE} = \frac{E^2}{2\pi^2} \Gamma_{\nu_R}(E). \quad (3)$$

In addition, the function $\Gamma_{\nu_R}(E)$ determines the spectral density of the energy luminosity of a supernova core via right-handed neutrinos:

$$\frac{dL_{\nu_R}}{dE} = V \frac{dn_{\nu_R}}{dE} E = V \frac{E^3}{2\pi^2} \Gamma_{\nu_R}(E). \quad (4)$$

Here, V is the volume of the neutrino-emitting region.

The function dL_{ν_R}/dE calculated in [10] is plotted in Fig. 1 for the neutrino magnetic moment $\mu_\nu = 3 \times 10^{-13} \mu_B$. On the one hand, this value is too small to affect the supernova dynamics. On the other hand, it is sufficiently large to provide the required luminosity level. In accordance with the existing supernova models (see, e.g., Fig. 11 in [14]), a significant part of the supernova core material has a fairly high temperature. For example, according to the model developed in [15], typical temperatures are 20-30 MeV. The model proposed in [16] predicts even higher temperatures. The energy distributions of the right-handed neutrino luminosity are plotted in Fig.1 for the temperatures $T = 35, 25, 15$, and 5 MeV, the electron and neutrino chemical potentials $\tilde{\mu}_e \simeq 300$ MeV and $\tilde{\mu}_{\nu_e} \simeq 160$ MeV, and the neutrino-emitting volume $V \simeq 4 \times 10^{18} \text{ cm}^3$.

To obtain a total energy of about 10^{51} erg extracted from the supernova central part by right-handed neutrinos in a time of about 0.2 s, the integral luminosity of these neutrinos should be about

$$L_{\nu_R} \simeq 4 \times 10^{51} \frac{\text{erg}}{\text{s}}. \quad (5)$$

An analysis shows that such luminosity can be generated by the considered process of neutrino-helicity conversion if the neutrino magnetic moment is not larger than refined upper limit (2) pointed out in [10]. The table illustrates the neutrino magnetic moments for which luminosity level (5) is achieved for any of the above-mentioned temperatures.

If the energy of right-handed neutrinos was converted into the energy of left-handed neutrinos, e.g., due to the well-known mechanism of spin oscillations, then an additional energy of about 10^{51} erg would be injected into the supernova envelope in a typical time of about a few tenths of a second.

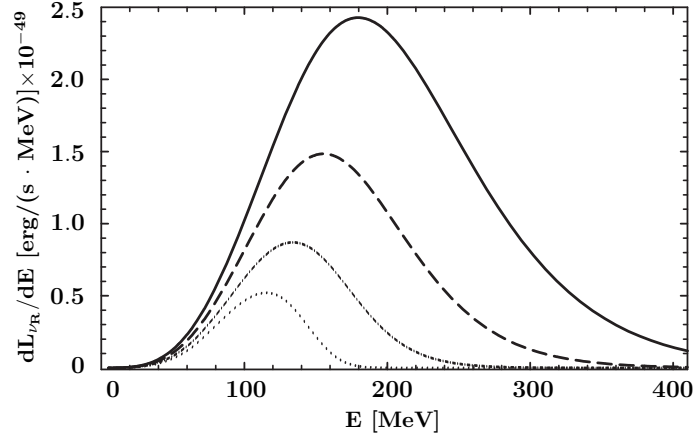


Figure 1: Energy distributions of the luminosity of right-handed neutrinos for plasma temperatures $T =$ (solid curve) 35, (dashed curve) 25, (dash-dotted curve) 15, and (dotted curve) 5 MeV and the neutrino magnetic moment $\mu_\nu = 3 \times 10^{-13} \mu_B$.

Table 1: Neutrino magnetic moments providing luminosity (5)

T (MeV)	$\mu_\nu / (10^{-12} \mu_B)$
35	0.29
25	0.42
15	0.64
5	0.97

It was noted above that the strong dominance of neutrino scattering by protons over scattering by electrons was not found in earlier studies. Hence, the possible number of the right-handed neutrinos generated in the collapse of the central region of a supernova was significantly underestimated.

At the same time, it is not evident that neutrino scattering by protons dominates over their scattering by plasma electrons. In this respect, we believe that a clear illustration of this dominance based on an analysis of a simplified case is quite expedient. The comparison of the typical parameters of the supernova core, where the temperature is $T \simeq 30$ MeV and the electron and neutrino chemical potentials are $\tilde{\mu}_e \simeq 300$ MeV and $\tilde{\mu}_{\nu_e} \simeq 160$ MeV, respectively, shows that the temperature is the smallest physical parameter. Hence, the limiting case of the completely degenerate plasma, $T = 0$, seems to yield a reasonable estimate. It is interesting that if the temperature is zero, the contributions from neutrino scattering by protons and electrons to the neutrino creation probability can be evaluated analytically using Eqs. (20) and (21) and the corresponding formulas from Appendix A in [10]. The contribution by ultrarelativistic electrons is given by the simple formula

$$\Gamma_{\nu_R}^{(e)}(E) = \frac{\mu_\nu^2 m_\gamma^2}{2\pi} (\tilde{\mu}_\nu - E) \theta(\tilde{\mu}_\nu - E), \quad (6)$$

where E is the energy of the generated right-handed neutrino, $m_\gamma^2 = 2\alpha \tilde{\mu}_e^2/\pi$ is the squared mass of a transverse plasmon, and $\tilde{\mu}_\nu$ is the neutrino chemical potential.

The analytical expression describing the proton contribution is somewhat more complicated since it depends additionally on the proton mass. The plasma electroneutrality condition for $T = 0$ takes the form $n_p = n_{e-}$ and ensures that the electron and proton Fermi momenta are the same: $k_F^{(e)} = k_F^{(p)}$. Then, the proton chemical potential coinciding with the Fermi energy is $\tilde{\mu}_p = E_F^{(p)} = \sqrt{m_p^2 + \tilde{\mu}_e^2}$ and the proton contribution is expressed in terms of the proton Fermi velocity $v_F = k_F^{(p)}/E_F^{(p)} = \tilde{\mu}_e/\tilde{\mu}_p = \tilde{\mu}_e/\sqrt{m_p^2 + \tilde{\mu}_e^2}$. As a result, the proton contribution is given by the expression

$$\Gamma_{\nu_R}^{(p)}(E) = \frac{\mu_\nu^2 m_\gamma^2 \tilde{\mu}_\nu}{2\pi} f_p(y), \quad y = \frac{E}{\tilde{\mu}_\nu}. \quad (7)$$

Here, the function $f_p(y)$ has the form

$$f_p(y) = \frac{1 + v_F/3}{1 - v_F} y, \quad (8)$$

for $0 \leq y \leq (1 - v_F)/(1 + v_F)$ and

$$f_p(y) = \frac{1 - y}{v_F} \theta(1 - y) \left[1 - \frac{(1 - v_F)^2}{12 y^2 v_F} (1 - y) (1 + 2y) \right]. \quad (9)$$

for $(1 - v_F)/(1 + v_F) \leq y \leq 1$. It is interesting that the integral contribution from protons is independent of the parameter v_F : $\int_0^1 f_p(y) dy = 1/2$.

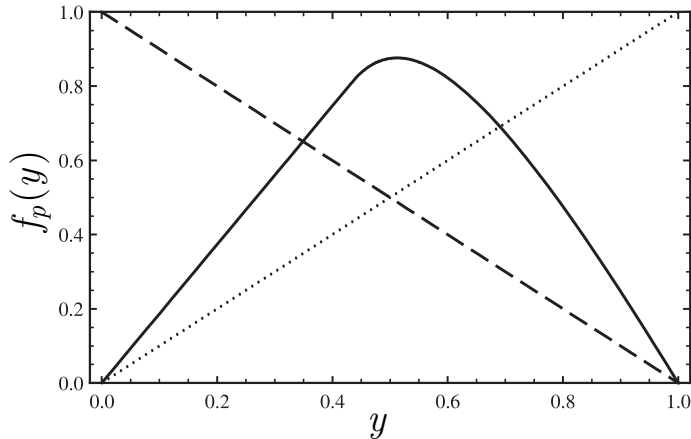


Figure 2: Plots of the function $f_p(y)$ for various v_F values. The dependence $f_e(y) = (1 - y)$ for the electron contribution is reproduced for $v_F = 1$ (dashed line). The value $v_F = 0.394$ (solid curve) corresponds to the effective proton mass $m_p \simeq 700$ MeV. The case $v_F = 0$ (dotted line) corresponds to the limit of infinitely large proton mass.

Note that the formal passage to the limit $m_p \rightarrow 0$ i.e., $v_F \rightarrow 1$ in Eqs. (7)–(9) yields $f_p(y) \rightarrow f_e(y) = (1 - y) \theta(1 - y)$, where the function $f_e(y)$ can be introduced in Eq. (6) in complete analogy with Eq. (7). Thus, as expected, Eq. (6) for the electron contribution is reproduced.

Figure 2 shows the plots of the function $f_p(y)$ for $v_F = 1, 0.394$, and 0. The value $v_F = 0.394$ corresponds to the effective proton mass $m_p \simeq 700$ MeV in a plasma with a nuclear density 3×10^{14} g/cm³ (see [3], p. 152). The value $v_F = 0$ corresponds to the formal limit $m_p \rightarrow \infty$ for which this function is also significantly simplified: $f_p(y) \rightarrow f_\infty(y) = y \theta(1 - y)$.

According to Eq. (4), the spectral density of the energy luminosity of the supernova core due to right-handed neutrinos is given by the formula

$$\frac{dL_{\nu_R}}{dE} = V \frac{\mu_\nu^2 m_\gamma^2 \tilde{\mu}_\nu^4}{4\pi^3} y^3 [f_e(y) + f_p(y)] . \quad (10)$$

The difference between the electron and proton contributions to the quantity given by Eq. (10) is illustrated in Fig. 3. It is clearly seen that the proton contribution to the luminosity is increased by the factor y^3 .

A comparison of Figs. 3 and 1 shows that allowance for a nonzero temperature results in a shift of the maximum of the energy distribution of the luminosity toward higher energies of right-handed neutrinos. This additionally enhances the proton contribution.

The flux of right-handed neutrinos from a collapsing supernova core enters the region of the envelope between the neutrinosphere of the radius R_ν and

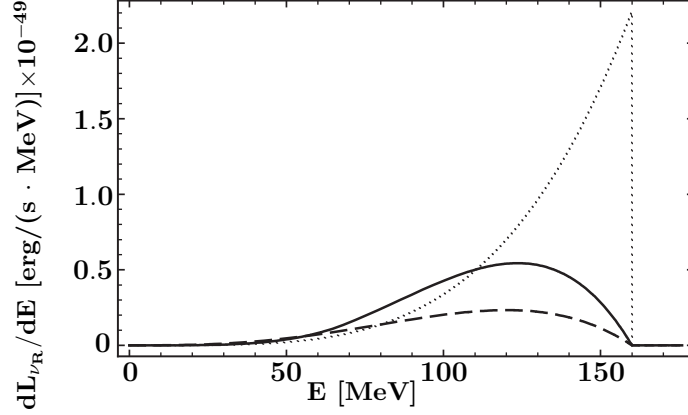


Figure 3: Energy distribution of the contributions from (dashed curve) electrons and protons with (solid curve) $m_p \simeq 700$ MeV and (dotted line) $m_p \rightarrow \infty$ to the luminosity of right-handed neutrinos for $T = 0$.

the shock-stagnation region of the radius R_s . According to commonly-accepted notions, the typical values of these quantities vary only slightly during the stagnation time and can be estimated as $R_\nu \sim 20\text{--}50$ km and $R_s \sim 100\text{--}200$ km. If a fairly strong magnetic field of $\sim 10^{13}$ G exists in the considered region, then neutrino spin oscillations occur and can be resonant under certain conditions.

The effect of the magnetic field on neutrinos with nonzero magnetic moments can be illustrated most conveniently using the equation for neutrino-helicity evolution in the uniform external magnetic field. The helicity-evolution equation taking into account the additional energy C_L gained by left-handed electron neutrinos in the matter can be written as [17–23]

$$i \frac{\partial}{\partial t} \begin{pmatrix} \nu_R \\ \nu_L \end{pmatrix} = \left[\hat{E}_0 + \begin{pmatrix} 0 & \mu_\nu B_\perp \\ \mu_\nu B_\perp & C_L \end{pmatrix} \right] \begin{pmatrix} \nu_R \\ \nu_L \end{pmatrix}, \quad (11)$$

where

$$C_L = \frac{3 G_F}{\sqrt{2}} \frac{\rho}{m_N} \left(Y_e + \frac{4}{3} Y_{\nu_e} - \frac{1}{3} \right). \quad (12)$$

Here, the ratio $\rho/m_N = n_B$ is the nucleon number density and $Y_e = n_e/n_B = n_p/n_B$, $Y_{\nu_e} = n_{\nu_e}/n_B$, n_{e,p,ν_e} are the number densities of electrons, protons, and neutrinos, respectively, B_\perp is the transverse magnetic-field component with respect to the direction of neutrino motion, and the term \hat{E}_0 proportional to the identity matrix is insignificant for our analysis.

Expression (12) for the additional energy of left-handed neutrinos is worthy of special analysis. It is important that this quantity can vanish in the considered region of the supernova envelope. In turn, this is a criterion for the resonance transition $\nu_R \rightarrow \nu_L$. Since the neutrino number density in the supernova envelope is fairly low, the quantity Y_{ν_e} in Eq. (12) is negligible. This yields

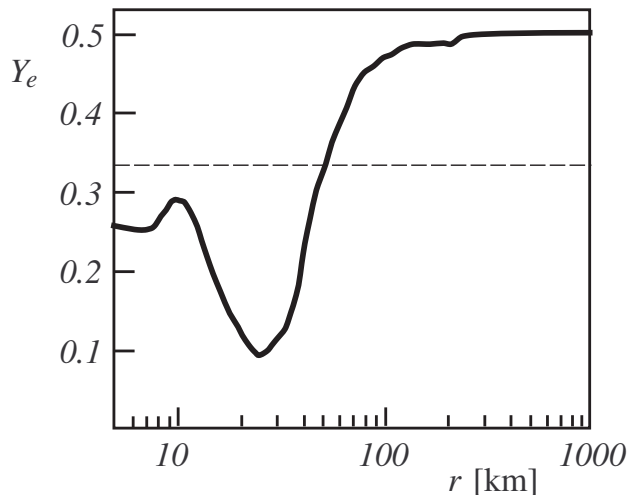


Figure 4: Qualitative behavior of the function $Y_e(r)$ at 0.1-0.2 s after the shock formation with a typical dip caused by a short-term neutrino burst (see, e.g., [4]). The dashed line corresponds to $Y_e = 1/3$.

the resonance condition in the form $Y_e = 1/3$. Note that Y_e in the supernova envelope is ~ 0.4 - 0.5 , which is typical of collapsing material. Nevertheless, the shock causing the dissociation of heavy nuclei makes the material more transparent to neutrinos. As a result, the so-called “short-term” neutrino burst is generated and the material in this region is significantly deleptonized. According to conventional notions, a characteristic dip down to ~ 0.1 is observed in the radial distribution of the quantity Y_e (see, e.g., [2, 4]). The qualitative behavior of $Y_e(r)$ is shown in Fig. 4. Thus, a point at which $Y_e = 1/3$ certainly exists. It is remarkable that only one such point with $dY_e/dr > 0$ exists (see [2, 4]).

Note that $Y_e = 1/3$ is a necessary, but insufficient condition for the resonant conversion of right-handed neutrinos into left-handed ones, $\nu_R \rightarrow \nu_L$. The so-called adiabatic condition should also be satisfied. The meaning of this condition is that the diagonal element C_L in Eq. (11) should be at least no larger than the off-diagonal element $\mu_\nu B_\perp$ if the distance from the resonance point is about the oscillation length. This leads to the criterion [22]

$$\mu_\nu B_\perp \gtrsim \left(\frac{dC_L}{dr} \right)^{1/2} \simeq \left(\frac{3 G_F}{\sqrt{2}} \frac{\rho}{m_N} \frac{dY_e}{dr} \right)^{1/2}. \quad (13)$$

The typical parameters of the considered region are as follows (see [2, 4]):

$$\frac{dY_e}{dr} \sim 10^{-8} \text{ cm}^{-1}, \quad \rho \sim 10^{10} \text{ g} \cdot \text{cm}^{-3}. \quad (14)$$

The magnetic field ensuring the resonance condition is

$$B_{\perp} \gtrsim 2.6 \times 10^{13} \text{ G} \left(\frac{10^{-13} \mu_B}{\mu_{\nu}} \right) \left(\frac{\rho}{10^{10} \text{ g} \cdot \text{cm}^{-3}} \right)^{1/2} \left(\frac{dY_e}{dr} \times 10^8 \text{ cm} \right)^{1/2}. \quad (15)$$

The mean free path with respect to beta-processes for a neutrino with an estimated energy of $E_{\nu} \sim 100\text{--}200 \text{ MeV}$ is

$$\lambda \simeq 800 \text{ m} \frac{1}{1 - Y_e} \left(\frac{150 \text{ MeV}}{E_{\nu}} \right)^2, \quad (16)$$

i.e., left-handed neutrinos are almost completely absorbed in the considered region.

Thus, our analysis shows that the Dar mechanism of the double conversion of neutrino helicity $\nu_L \rightarrow \nu_R \rightarrow \nu_L$ exists under the following not very severe conditions: the Dirac-neutrino magnetic moment should be in the range $10^{-13} \mu_B < \mu_{\nu} < 10^{-12} \mu_B$ and a magnetic field of $\sim 10^{13} \text{ G}$ should exist in the region $R_{\nu} < R < R_s$. In this case, an additional energy of about

$$\Delta E \simeq L_{\nu_R} \Delta t \sim 10^{51} \text{ erg}, \quad (17)$$

is injected into this region during the shock-stagnation time $\Delta t \sim 0.2\text{--}0.4 \text{ s}$. This energy is sufficient to solve the problem.

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